Chern-Simons coefficient in supersymmetric non-Abelian Chern-Simons Higgs theories

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By taking into account the effect of the would be Chern-Simons term, we calculate the quantum correction to the Chern-Simons coefficient in supersymmetric Chern-Simons Higgs theories with matter fields in the fundamental representation of SU(n). Because of supersymmetry, the corrections in the symmetric and Higgs phases are identical. In particular, the correction is vanishing for $N=3$ supersymmetric Chern-Simons Higgs theories. The result should be quite general, and have important implications for the more interesting cases when the Higgs field is in the adjoint representation. [S0556-2821(99)03916-8]

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Chern-Simons theories can give rise to particle excitations with fractional spin and statistics, and thus have been used as effective field theories to study the fractional quantum Hall effect [1–3]. They are also interesting when the Higgs fields with a special sixth-order potential are included so that the systems admit a Bogomol'nyi bound in energy [4]. The bound is saturated by solutions satisfying a set of first-order self-duality equations [5]. These solutions have a rich structure and have been under extensive study especially when the gauge symmetry is non-Abelian with the Higgs field in the adjoint representation [6]. It is known that the self-duality in these systems signifies an underlying $N=2$ supersymmetry and thus the Bogomol'nyi bound is expected to be preserved in the quantum regime [7]. Furthermore, when these theories are dimensionally reduced, an additional Noether charge appears, which in turns yields a Bogomol'nyi-Prasad-Sommerfield- type domain wall [8].

The quantum correction to the Chern-Simons coefficient has also attracted a lot of attention. For theories without massless charged particles and the gauge symmetry not spontaneously broken, Coleman and Hill have shown in the Abelian case that only the fermion one-loop diagram can contribute to the correction to the Chern-Simons coefficient and yields $1/4\pi$ [9]. The quantization of the correction can be understood with a topological argument in the spinor space by making use of the Ward-Takahashi identity [10]. When there is spontaneous breaking of gauge symmetry, one can show that there exists in the effective action the so-called would be Chern-Simons terms, which induces terms similar to the Chern-Simons terms in the Higgs phase [11]. By taking into account the effect of the would be Chern-Simons term, it has been shown that the one-loop correction in the Higgs phase is identical to that in the symmetric phase [12]. On the other hand, if the charged particles are massless, both scalars and spinors can contribute to the correction at the two-loop level, and it is not quantized [13].

The situation becomes even more intriguing when the gauge symmetry is non-Abelian: the Chern-Simons coefficient must be an integer multiple of $1/4\pi$ for the systems to be invariant under large gauge transformation; otherwise the theories are not quantum mechanically consistent. Therefore, it is interesting to confirm that the quantization condition is not spoiled by quantum effects. In the symmetric phase, this has been shown to one loop [14]. When there is no bare Chern-Simons term, it is also verified up to two loops considering only the fermionic contribution [15]. In the Higgs phase, it has been known for some time that if there is remaining symmetry, e.g., SU(n) with $n\geqslant 3$, the quantization condition will still be satisfied [16–18]. However, if the gauge symmetry is completely broken, e.g., SU(2), a simple-minded calculation shows that the correction is again complicated and not quantized [11]. Although one may argue that this arises because there is no well-defined symmetry generator in such a case, a better way to understand the whole thing is again to note the effect of the would be Chern-Simons terms. They are invariant even under large gauge transformation, and their coefficients need not be quantized. Therefore, we must subtract out their contribution to obtain the correct result. Indeed, more careful calculation shows that for the Higgs being in fundamental SU(n) the quantization condition is always satisfied whether the gauge symmetry is completely broken or not [19]. As a result, a more or less unifying picture of the quantum correction to the Chern-Simons coefficient has emerged.

In pure non-Abelian Chern-Simons theories, there is also the so-called regularization dependence of the quantum corrections to the Chern-Simons coefficient:

$$\Delta\kappa=\mathrm{sgn}(C_v),$$

if we introduce the Yang-Mills term as a UV regulator, while

$$\Delta\kappa=0,$$

if we do not [20]. Here, C_v is the the quadratic Casimir operator in the adjoint representation of the gauge group. Further studies suggest that every local regulator manifestly preserving Becchi-Rouet-Stora invariance and unitarity would give rise to the same quantum correction [21]. Interestingly, it has been shown that $N=1$ supersymmetric Yang-Mills-Chern-Simons theory is finite to all orders [22]. Moreover, if the regulator is supersymmetric, the corrections become regularization independent [23]. In particular, the corrections are vanishing for $N=2,3$ supersymmetric Chern-

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Simons theories. Hence, we would like to know what will happen if there is also spontaneous breaking of gauge symmetry in the system.

In this paper, we calculate the quantum corrections to the Chern-Simons coefficient in supersymmetric Chern-Simons Higgs theories with the Higgs being in the fundamental SU(n). It turns out that the result is partially regularization dependent. If we do not introduce the Yang-Mills term, the quantum corrections are quantized and identical in the symmetric and Higgs phase because of supersymmetry. On the other hand, if we do, the result is more complicated. For $n \geqslant 3$, the quantum corrections are still identical in the two phases. For $n=2$, however, the quantum corrections become different in the two phases. We conclude with some comments on its implication and possible future direction.

With matter fields in the fundamental SU(n), the $N=3$ supersymmetric non-Abelian Chern-Simons Higgs theories can be simplified to [24]

$$\begin{aligned}\mathcal{L}=&-i\kappa\epsilon^{\mu\nu\rho}\mathrm{tr}\left\{A_\mu\partial_\nu A_\rho-\frac{2}{3}iA_\mu A_\nu A_\rho\right\}+|D_\mu\phi_1|^2+|D_\mu\Phi_2|^2+\overline{\psi}\gamma^\mu D_\mu\psi+\overline{\chi}\gamma^\mu D_\mu\chi+\frac{1}{\kappa^2}(|\phi_1|^2+|\Phi_2|^2)\\&\times\left\{\left[\frac{(n-1)}{2n}(|\phi_1|^2-|\Phi_2|^2)+v^2\right]^2+\frac{1}{4}|\phi_1|^2|\Phi_2|^2+\frac{(3n-2)(n-2)}{4n^2}|\phi_1^\dagger\Phi_2|^2\right\}+\frac{1}{\kappa}\left\{\left[v^2-\frac{1}{2n}|\phi_1|^2\right.\right.\\&\left.-\frac{(2n-1)}{2n}|\Phi_2|^2\right]\overline{\psi}\psi+\left[-v^2-\frac{(2n-1)}{2n}|\phi_1|^2-\frac{1}{2n}|\Phi_2|^2\right]\overline{\chi}\chi\Bigg\}+\frac{1}{2}[(\overline{\psi}\phi_1)(\phi_1^\dagger\psi)+(\overline{\chi}\Phi_2)(\Phi_2^\dagger\chi)]\\&-\frac{(n-2)}{2n}[(\overline{\psi}\Phi_2)(\Phi_2^\dagger\psi)+(\overline{\chi}\phi_1)(\phi_1^\dagger\chi)]+\frac{(n-1)}{2n}[(\overline{\psi}\phi_1)(\psi^\dagger\phi_1)+(\phi_1^\dagger\overline{\psi}^\dagger)(\phi_1^\dagger\psi)+(\overline{\chi}\Phi_2)(\chi^\dagger\Phi_2)+(\Phi_2^\dagger\overline{\chi}^\dagger)(\Phi_2^\dagger\chi)]\\&-\frac{(n-1)}{2n}[(\Phi_2^\dagger\phi_1)(\overline{\psi}\chi)+(\phi_1^\dagger\Phi_2)(\overline{\chi}\psi)+(\overline{\psi}\phi_1)(\Phi_2^\dagger\chi)+(\overline{\chi}\Phi_2)(\Phi_2^\dagger\chi)]-[(\overline{\psi}\phi_1)(\chi^\dagger\Phi_2)+(\Phi_2^\dagger\overline{\chi}^\dagger)(\phi_1^\dagger\psi)]\\&+\frac{1}{n}[(\overline{\psi}\Phi_2)(\chi^\dagger\phi_1)+(\phi_1^\dagger\overline{\chi}^\dagger)(\Phi_2^\dagger\psi)].\end{aligned}\tag{1}$$

Here $D_\mu=(\partial_\mu-iA_\mu^m T^m)$ and $\gamma_\mu=\sigma_\mu$ so that the gamma matrices satisfy $\gamma_\mu\gamma_\nu=\delta_{\mu\nu}+i\epsilon_{\mu\nu\rho}\gamma_\rho$, with $\epsilon_{012}=1$. The generators satisfy $[T^m,T^n]=if^{lmn}T^l$, with the normalization $\mathrm{tr}\{T^mT^n\}=\delta^{mn}/2$ and $\Sigma_m(T^m)_{\alpha\beta}(T^m)_{\gamma\delta}=\frac{1}{2}\delta_{\alpha\delta}\delta_{\beta\gamma}-(1/2n)\delta_{\alpha\beta}\delta_{\gamma\delta}$.

We will use the background field gauge so that the effective action is explicitly gauge invariant and the gauge fields do not get renormalized. This can be done by separating A_μ into the background part A_μ and the quantum part Q_μ. In the Higgs phase, $\Phi_2=\phi_2+\varphi$ with $\varphi^\dagger\varphi=|\varphi|^2$. As usual, the gauge fixing and the Faddeev-Popov (FP) ghost terms are given by

$$\mathcal{L}_{gf}=\frac{1}{2\xi}\{(\hat{D}_\mu Q_\mu)^m+i\xi(\varphi^\dagger T^m\phi_2-\phi_2^\dagger T^m\varphi)\}^2\tag{2}$$

and

$$\mathcal{L}_{\mathrm{FP}}=2\,\mathrm{tr}\{(\hat{D}_\mu\overline{\eta})(\hat{D}_\mu\eta)-i(\hat{D}_\mu\overline{\eta})[Q_\mu,\eta]\}+\xi(\varphi^\dagger\overline{\eta}\eta\varphi-\varphi^\dagger\eta\overline{\eta}\varphi)+\xi(\varphi^\dagger\overline{\eta}\eta\phi_2-\phi_2^\dagger\eta\overline{\eta}\varphi).\tag{3}$$

Here $\hat{D}_\mu$ is the covariant derivative using the background field. Combining Eqs. (1), (2), and (3), we see the relevant quadratic terms are

$$\begin{aligned}\mathcal{L}_0=&\frac{1}{2}Q_\mu^m\left\{[i\kappa\epsilon_{\mu\nu\rho}\partial_\rho]\delta_{mn}-\frac{1}{\xi}\partial_\mu\partial_\nu+\delta_{\mu\nu}[(\varphi^\dagger T^mT^n\varphi)+(\varphi^\dagger T^nT^m\varphi)]\right\}Q_\nu^n\\&+\frac{1}{2}(\phi_2^\dagger,\phi_2^T)\begin{pmatrix}-\partial^2+\frac{(n-1)^2|\varphi|^2\varphi\varphi^\dagger}{2n^2\kappa^2}+\frac{\xi|\varphi|^2}{2}-\frac{\xi\varphi\varphi^\dagger}{2n} & \frac{(n-1)^2|\varphi|^2\varphi\varphi^T}{2n^2\kappa^2}-\frac{(n-1)\xi\varphi\varphi^T}{2n}\\ \frac{(n-1)^2|\varphi|^2\varphi^*\varphi^\dagger}{2n^2\kappa^2}-\frac{(n-1)\xi\varphi\varphi^\dagger}{2n} & -\partial^2+\frac{(n-1)^2|\varphi|^2\varphi^*\varphi^T}{2n^2\kappa^2}+\frac{\xi|\varphi|^2}{2}-\frac{\xi\varphi^*\varphi^T}{2n}\end{pmatrix}\begin{pmatrix}\phi_2\\ \phi_2^*\end{pmatrix}\\&+\frac{1}{2}(\overline{\psi},\overline{\psi}^*)\begin{pmatrix}\gamma\cdot\partial-\frac{|\varphi|^2}{2\kappa}-\frac{(n-2)\varphi\varphi^\dagger}{2n\kappa} & 0\\ 0 & \gamma\cdot\partial-\frac{|\varphi|^2}{2\kappa}-\frac{(n-2)\varphi^*\varphi^T}{2n\kappa}\end{pmatrix}\begin{pmatrix}\psi\\ \psi^*\end{pmatrix}\end{aligned}$$

$$+\frac{1}{2}(\overline{\chi},\overline{\chi^*})\begin{pmatrix}\gamma\cdot\partial-\frac{|\varphi|^2}{2\kappa}+\frac{\varphi\varphi^\dagger}{2\kappa} & \frac{(n-1)\varphi\varphi^T}{n\kappa}\\ \frac{(n-1)\varphi^*\varphi^\dagger}{n\kappa} & \gamma\cdot\partial-\frac{|\varphi|^2}{2\kappa}+\frac{\varphi^*\varphi^T}{2\kappa}\end{pmatrix}\begin{pmatrix}\chi\\ \chi^*\end{pmatrix}+f^{lmn}\left\{\frac{1}{\xi}(\partial_\mu Q^l_\mu)A^m_\mu Q^n_\nu-\frac{i\kappa}{2}\epsilon_{\mu\nu\rho}A^l_\mu Q^m_\nu Q^n_\rho\right\}$$

$$+2(\varphi^\dagger A_\mu Q_\mu\phi_2)+2(\phi_2^\dagger Q_\mu A_\mu\varphi)-i\overline{\psi}\gamma^\mu A_\mu\psi-i\overline{\chi}\gamma^\mu A_\mu\chi. \tag{4}$$

In our case, there are two relevant would-be Chern-Simons terms:

$$O_1=\epsilon^{\mu\nu\rho}i\{\Phi_2^\dagger T^m(D_\mu\Phi_2)-(D_\mu\Phi_2)^\dagger T^m\Phi_2\}F^m_{\nu\rho},$$

$$O_2=\epsilon^{\mu\nu\rho}i\{\Phi_2^\dagger(D_\mu\Phi_2)-(D_\mu\Phi_2)^\dagger\Phi_2\}(\Phi_2^\dagger F_{\nu\rho}\Phi_2). \tag{5}$$

In the Higgs phase, they give rise to

$$\epsilon^{\mu\nu\rho}A^n_\mu F^m_{\nu\rho}\{(\varphi^\dagger T^m T^n\varphi)+(\varphi^\dagger T^n T^m\varphi)\},$$

$$2\epsilon^{\mu\nu\rho}A^n_\mu F^m_{\nu\rho}(\varphi^\dagger T^m\varphi)(\varphi^\dagger T^n\varphi), \tag{6}$$

respectively. We note that their transformation property under the SU(n) symmetry are different from the quadratic part of the Chern-Simons term. Therefore, we will leave the vacuum expectation value φ in the general form so that it is easier to extract the correction to the Chern-Simons coefficient. For this purpose, we express the propagators in terms of the following projection operators:

$$(P_1)_{mn}=\delta_{mn}-2[(\hat\varphi^\dagger T^m T^n\hat\varphi)+(\hat\varphi^\dagger T^n T^m\hat\varphi)]+\frac{2(n-2)}{(n-1)}(\hat\varphi^\dagger T^m\hat\varphi)(\hat\varphi^\dagger T^n\hat\varphi),$$

$$(P_2)_{mn}=2[(\hat\varphi^\dagger T^m T^n\hat\varphi)+(\hat\varphi^\dagger T^n T^m\hat\varphi)]-4(\hat\varphi^\dagger T^m\hat\varphi)\times(\hat\varphi^\dagger T^n\hat\varphi),$$

$$(P_3)_{mn}=\frac{2n}{(n-1)}(\hat\varphi^\dagger T^m\hat\varphi)(\hat\varphi^\dagger T^n\hat\varphi);$$

$$Q_1=\begin{pmatrix}I-\hat\varphi\hat\varphi^\dagger & 0\\ 0 & I-\hat\varphi^*\hat\varphi^T\end{pmatrix},$$

$$Q_2=\frac{1}{2}\begin{pmatrix}\hat\varphi\hat\varphi^\dagger & \hat\varphi\hat\varphi^T\\ \hat\varphi^*\hat\varphi^\dagger & \hat\varphi^*\hat\varphi^T\end{pmatrix},$$

$$Q_3=\frac{1}{2}\begin{pmatrix}\hat\varphi\hat\varphi^\dagger & -\hat\varphi\hat\varphi^T\\ -\hat\varphi^*\hat\varphi^\dagger & \hat\varphi^*\hat\varphi^T\end{pmatrix}, \tag{7}$$

where $\hat\varphi\equiv\varphi/|\varphi|$.

With these projection operators, it is now straightforward to obtain the propagators of Q_μ,ϕ_2,ψ, and χ:

$$\Delta^{mn}_{\mu\nu}(k)=\{[\Delta^1_{\mu\nu}(k)](P_1)_{mn}+[\Delta^2_{\mu\nu}(k)](P_2)_{mn}+bigl[\Delta^3_{\mu\nu}(k)](P_3)_{mn}\},$$

$$D(k)=\{[D^1(k)]Q_1+[D^2(k)]Q_2+[D^3(k)]Q_3\}, \tag{8}$$

$$S_\psi(k)=\{[S^1(k)]Q_1+[S^3(k)]Q_2+[S^3(k)]Q_3\},$$

$$S_\chi(k)=\{[S^1(k)]Q_1+[S^2(k)]Q_2+[S^3(k)]Q_3\}.$$

Here,

$$\Delta^1_{\mu\nu}(k)=\frac{\epsilon_{\mu\nu\rho}k^\rho}{\kappa k^2}+\frac{\xi k_\mu k_\nu}{k^4},$$

$$\Delta^2_{\mu\nu}(k)=\frac{M_W(\delta_{\mu\nu}-k_\mu k_\nu/k^2)+\epsilon_{\mu\nu\rho}k^\rho}{\kappa(k^2+M_W^2)}+\frac{\xi k_\mu k_\nu}{k^2\left(k^2+\frac{1}{2}\xi|\varphi|^2\right)},$$

$$\Delta^3_{\mu\nu}(k)=\frac{M_Z(\delta_{\mu\nu}-k_\mu k_\nu/k^2)+\epsilon_{\mu\nu\rho}k^\rho}{\kappa(k^2+M_Z^2)}+\frac{\xi k_\mu k_\nu}{k^2\left[k^2+\frac{(n-1)}{n}\xi|\varphi|^2\right]},$$

$$D^1(k)=\frac{1}{\left(k^2+\frac{1}{2}\xi|\varphi|^2\right)},$$

$$D^2(k)=\frac{1}{(k^2+M_Z^2)}, \tag{9}$$

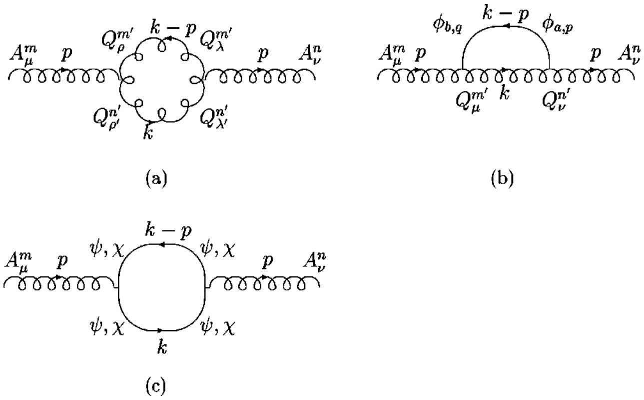

FIG. 1. The one-loop diagrams that contribute to the parity odd part of the vacuum polarization. (a) involves an internal gluon loop, (b) involves an internal loop with both gluon and Higgs fields, and (c) involves an internal fermion loop.

$$D^3(k)=\frac{1}{[k^2+(n-1)/n\xi|\varphi|^2]},$$

$$S^1(k)=\frac{1}{(i\gamma\cdot k-M_W)},$$

$$S^2(k)=\frac{1}{(i\gamma\cdot k+M_Z)},$$

$$S^3(k)=\frac{1}{(i\gamma\cdot k-M_Z)},$$

where $M=\kappa g^2$, $M_W=|\varphi|^2/(2\kappa)$, and $M_Z=(n-1)|\varphi|^2/(n\kappa)$.

To determine the renormalization of the Chern-Simons coefficient, it is sufficient to calculate the parity odd part of the vacuum polarization. The three relevant diagrams are shown in Fig. 1: one with a gluon loop, one with a gluon-Higgs loop, and one with a fermion loop [18]. After some algebra, we see that the vacuum polarization can be decomposed into three parts:

$$[\Pi^{mn}_{\mu\nu}(p)]_{\rm odd}=\epsilon_{\mu\nu\rho}p_\rho\{\Pi_1(p^2)\delta_{mn}+\Pi_2(p^2)[(\hat{\varphi}^\dagger T^m T^n\hat{\varphi})+(\hat{\varphi}^\dagger T^n T^m\hat{\varphi})]+\Pi_3(p^2)(\hat{\varphi}^\dagger T^m\hat{\varphi})(\hat{\varphi}^\dagger T^n\hat{\varphi})\}. \tag{10}$$

Since the two would-be Chern-Simons terms only contribute to $\Pi_2(0)$ and $\Pi_3(0)$, we only need to calculate $\Pi_1(0)$ to find the correction to the Chern-Simons coefficient. In the Landau gauge,

$$\Pi_1(p)=\Pi_B(p)+2\Pi_F(p), \tag{11}$$

with

$$\begin{aligned}\Pi_B(p)=&\int\frac{d^3k}{(2\pi)^3}\left\{\frac{[k^2p^2-(k\cdot p)^2]}{p^2(k^2+M_W^2)[(k-p)^2+M_W^2]}\right\}\\&\times\left\{\frac{-M_W}{2(k-p)^2}+\frac{-M_W}{2(k)^2}\right\}\\&+\int\frac{d^3k}{(2\pi)^3}\left\{\frac{-M_W(k\cdot p)}{p^2(k^2+M_W^2)(k-p)^2}\right.\\&\left.+\frac{M_W[-k^2p^2-(k\cdot p)^2+2k^2(k\cdot p)]}{p^2k^2(k^2+M_W^2)(k-p)^2}\right\},\end{aligned} \tag{12}$$

$$\Pi_F(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M_W}{(k^2+M_W^2)[(k-p)^2+M_W^2]}\right\}.$$

In the zero momentum limit,

$$\Pi_B(0)=\frac{-\kappa}{4\pi|\kappa|}, \tag{13}$$

$$\Pi_F(0)=\frac{\kappa}{8\pi|\kappa|}.$$

By throwing away ϕ_1 and χ, we can also obtain the correction for $N=2$ supersymmetric Chern-Simons Higgs theories. In sum, the corrections are

$$\Delta\kappa_{\rm N=3}=0, \tag{14}$$

$$\Delta\kappa_{\rm N=2}=\frac{-\kappa}{8\pi|\kappa|}. \tag{15}$$

Both results are identical to those in the symmetric phase. Therefore, the degeneracy between the symmetric and asymmetric vacua is preserved as we have expected for supersymmetric theories. This is confirmed by calculating the effective potential of ϕ_2.

The situation is quite different, if we introduce the Yang-Mills term as an ultraviolet regulator. From the result in Ref. [19], we have

$$\Pi_B(p)=\frac{(n-1)}{2}\Pi^{Ia}(p)+\frac{1}{2}\Pi^{Ib}(p), \tag{16}$$

in the Landau gauge. Here,

$$\Pi^{Ia}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M[k^2p^2-(k\cdot p)^2][4M^2+10k^2-10k\cdot p+8p^2]}{p^2k^2(k^2+M^2)(k-p)^2[(k-p)^2+M^2]}\right\}+\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M[-2k^2p^2-2(k\cdot p)^2+4p^2(k\cdot p)]}{p^2k^2(k^2+M^2)(k-p)^2}\right\},$$

$$\Pi^{Ib}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M[k^2p^2-(k\cdot p)^2]}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)[(k-p)^2+M_{W^+}^2][(k-p)^2+M_{W^-}^2]}\right\}$$

$$\times\left\{6M^2+\frac{(k^2+M_{W^+}M_{W^-})[-M^2+8k^2-4k\cdot p+4p^2]}{k^2}+\frac{[(k-p)^2+M_{W^+}M_{W^-}][-M^2+8k^2-12k\cdot p+8p^2]}{(k-p)^2}\right.$$

$$\left.+\frac{(k^2+M_{W^+}M_{W^-})[(k-p)^2+M_{W^+}M_{W^-}][-6k^2+6k\cdot p-4p^2]}{k^2(k-p)^2}\right\}$$

$$+\int\frac{d^3k}{(2\pi)^3}\left\{\frac{-2M(k\cdot p)[M_{W^+}M_{W^-}+2k^2-2p^2]}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2}+\frac{M(k^2+M_{W^+}M_{W^-})[-2k^2p^2-2(k\cdot p)^2+4k^2(k\cdot p)]}{p^2k^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2}\right\}. \tag{17}$$

They come from the unbroken and broken sectors, respectively. In the zero momentum limit, we see

$$\Pi^{Ia}(0)=\frac{\kappa}{2\pi|\kappa|},$$

$$\Pi^{Ib}(0)=0. \tag{18}$$

It is obvious that taking the limit that $g\to\infty$ does not change the above result.

For $n\geqslant 3$, there is remaining gauge symmetry and

$$\Pi_B(0)=\frac{(n-1)\kappa}{4\pi|\kappa|}. \tag{19}$$

Consequently,

$$\Delta\kappa_{\mathrm{N}=3}=\frac{n\kappa}{4\pi|\kappa|},$$

$$\Delta\kappa_{\mathrm{N}=2}=\frac{(n-1/2)\kappa}{4\pi|\kappa|}. \tag{20}$$

Again, both the above results are identical to those in the symmetric phase. In the SU(2) case the gauge symmetry is completely broken, and there is no such thing as an unbroken part in the Higgs phase. As a result, the first terms in Eq. (16) should not have been there. Since $\Pi^{Ib}(0)=0$ and thus the bosonic part is vanishing, the quantum correction to the Chern-Simons coefficient comes from the fermionic part only and is

$$\Delta\kappa_{\mathrm{N}=3}=\frac{\kappa}{4\pi|\kappa|},$$

$$\Delta\kappa_{\mathrm{N}=2}=\frac{\kappa}{8\pi|\kappa|}. \tag{21}$$

Both results are different from those in the symmetric phase. This indicates that the supersymmetry is broken when the gauge group is fundamental SU(2). Since the Yang-Mills term itself does not respect supersymmetry, it is hardly surprising. The confusing part is why this happens only for the SU(2) case. One possible way to clarify the above confusion is to do a derivative expansion-type calculation as in Ref. [12].

The results that the quantum correction to the Chern-Simons coefficient in supersymmetric Chern-Simons Higgs theories are identical in the symmetric and Higgs phases is interesting and have important implications. It is well known that non-Abelian self-dual Chern-Simons Higgs theories, with the Higgs in the adjoint representation, have rich vacuum structure. It has been quite a challenge to verify that the quantum correction to the Chern-Simons coefficient is quantized in these systems. If the results obtained above can be generalized to the adjoint representation, we can calculate the quantum correction in self-dual Chern-Simons Higgs theories by calculating the fermionic part in the corresponding supersymmetric Chern-Simons Higgs theories. Finally, although the calculation done here is only for supersymmtric Chern-Simons Higgs theories, we believe the results also apply to supersymmetric Yang-Mills Chern-Simons Higgs theories based on our experience from Ref. [23].

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